

**Hale School**

**MATHEMATICS**

**SPECIALIST**

**3CD**

**Semester Two Examination 2011**

**MARKING KEY and SOLUTIONS**

**Section One**

**Calculator-Free**

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**Question 1 [9 marks]**

Give exact values for the following :

(a) 

[1]

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| **Solution** |
|   = cis(-π/2) = - i  |
| **Specific Behaviours**  |
| ✓ Correct answer |

(b) 

[2]

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| --- |
| **Solution** |
|   =  = cis(-π) = -1  |
| **Specific Behaviours**  |
| ✓ Recognises cis with the correct argument✓ Uses DeMoivre’s Theorem to multiply argument correctly and give the correct answer |

(c) 

[3]

|  |
| --- |
| **Solution** |
|   =  =  =   |
| **Specific Behaviours**  |
| ✓ Recognises the limit as a derivative✓ Correct identification of the function cos 2x✓ Determines the exact value |

(d) The shaded area under the curve y = cos 2x.

[3]

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| **Solution** |
|   square units  |
| **Specific Behaviours**  |
| ✓ Correct expression for the area✓ Anti-differentiates correctly✓ Correct evaluation |

**Question 2 [9 marks]**

Given that z = eix and w = e–ix (where x is a real number) :

(a) express cis(3x) w in terms of z.

[2]

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| **Solution** |
|    |
| **Specific Behaviours**  |
| ✓ Expresses cis(3x) in terms of a complex exponential✓ Correct expression in terms of z |

(b) if z - w is expressed in the form a + bi determine the values of a and b.

[2]

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| **Solution** |
|    Hence a = 0, b = 2 sin x |
| **Specific Behaviours**  |
| ✓ Recognises expression to give twice the imaginary part✓ Correct values for both a and b. |

(c) simplify z3 + w3

[2]

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| **Solution** |
|    |
| **Specific Behaviours**  |
| ✓ Correct use of index laws with the complex exponential✓ Simplifies correctly in terms of twice the real part |

(d) solve for x given that z4 + 1 = 0

[3]

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| **Solution** |
|       Hence x =  ,  ,  ,  |
| **Specific Behaviours**  |
| ✓ Expresses -1 in polar form with argument π✓ Correct expression for the 4 solutions in polar form✓ Correct values for x (using convention between -π to π) |

**Question 3 [6 marks]**

Points A, B and C have respective position vectors given by :

 **a** = **i** + **j**  - **k**

 **b** = **i** + **j**  + **k**

 **c** = 2**i** + **j**

Determine :

(a) the value of cosine of the angle between vectors **a** and **b**.

[2]

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| **Solution** |
|  **a . b**   1 = 3 cos θ Hence cos θ =  |
| **Specific Behaviours**  |
| ✓ Correct use of dot product and the magnitudes of each vector✓ Correct value for cos θ |

(b) the vector equation of the line containing points A and B.

[2]

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| **Solution** |
|  Direction vector **d = b - a** =  Vector equation line **r** =   |
| **Specific Behaviours**  |
| ✓ Finds an appropriate direction vector✓ Expresses in correct point-direction vector form (does not have to express as a single vector) |

(c) the vector equation of the plane containing vectors **a** and **b** and also containing the point C.

[2]

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| **Solution** |
|  Vector equation plane **r** =   Alternative answer **r .**  |
| **Specific Behaviours**  |
| ✓ Uses 2 parameters with the direction vectors a and b✓ Expresses in correct vector form (does not have to express as a single vector) NO MARKS if students uses vectors a and b as points in the plane |

 **Question 4 [3 marks]**

Evaluate the definite integral  exactly :

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| **Solution** |
|    |
| **Specific Behaviours**  |
| ✓ Correct use of the cosine DOUBLE angle identity✓ Anti-differentiates correctly✓ Correct evaluation NO marks for use of sin3x as the anti-derivative |

**Question 5 [4 marks]**

(a) Determine matrix T = 

[1]

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| **Solution** |
|  T =   |
| **Specific Behaviours**  |
| ✓ ALL Matrix elements are correct |

(b) Hence if matrix T represents a transformation matrix, describe the actions of matrix T.

[3]

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| **Solution** |
|  T = A B i.e. B then A i. Reflect about the line y = -x and then ii. Dilate horizontally about x = 0 with factor 2   |
| **Specific Behaviours**  |
| ✓ Description of the reflection matrix✓ Description of the dilation matrix✓ Correct order i.e. reflect then dilate |

**Question 6 [5 marks]**

The natural logarithm function can be defined as ln(x) =  where x > 0.

(a) Given that a, b > 0, using the substitution u =  find an expression for the definite integral  .

[3]

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| **Solution** |
|   |
| **Specific Behaviours**  |
| ✓ Changes limits correctly✓ Expresses integrand correctly✓ Recognises answer using the natural logarithm definition |

(b) By considering  =  +  and using the result from part (a) make a deduction about the natural logarithm function.

[2]

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| **Solution** |
|   =  +   ln (ab) = ln a + ln b |
| **Specific Behaviours**  |
| ✓ Uses the result from part (a)✓ Deduces that the log(Product) = sum of logarithms |

**Question 7 [4 marks]**

Prove, by any method, that the cube of any number that is 2 more than a multiple of 3 is always 1 less than a multiple of 9.

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| **Solution** |
|  Let n be any counting number. Hence 3n + 2 is two more than a multiple of 3 (the particular number) Consider (3n + 2)3 = (3n)3 + 3(3n)2(2) + 3(3n)(22) + 23 = 27n3 + 54n2 + 36n + 8 = 27n3 + 54n2 + 36n + 9 - 1 = 9(3n3 + 6n2 + 4n + 1) - 1 Hence (3n + 2)3 is always of the form 9k - 1  |
| **Specific Behaviours**  |
| ✓ Express the cube of the particular number✓ Expand correctly the cube of the binomial✓ Simplify each term correctly✓ Express in the form 9k - 1 |